Dispersion management for randomly varying optical fibers

Mark J. Ablowitz and Jamison T. Moeser

Department of Applied Mathematics, University of Colorado, Boulder, Colorado 80309-0526

Received December 8, 2003

An asymptotic theory for optical pulse propagation in a dispersion-managed (DM) fiber with random dispersion is presented. The validity of the theory is verified with direct numerical simulation. The equations that describe the slow evolution of initial pulses have special solutions that, for fibers with moderate noise in the dispersion profile, perform much better than ideal DM solitons optimized for the unperturbed fiber. © 2004 Optical Society of America

OCIS codes: 060.4510, 190.5530.

The field of optical fiber communications has developed impressively in recent years, and long-haul high-speed transmission over transoceanic distances with total capacity in excess of terabits per second has already been achieved. In complex transmission systems, signal deformation caused by various dispersive and nonlinear properties of optical fibers can become major limitations, and fundamental studies of signal transmission in optical fibers have and will continue to play a critical role in optimizing system performance.

In a dispersion-managed (DM) system[1] the fiber is composed of spans with group-velocity dispersion (GVD) values that alternate in sign. The spans are assembled in such a manner as to create a transmission line with high local and low average GVD. This mechanism has been successful, and dispersion management is widely employed in current technology. In certain operating regimes there are special solutions for the equations governing the pulse dynamics in a DM fiber. These solutions, termed DM solitons, are stable, stationary solutions to the DM nonlinear Schrödinger (DMNLS) equation, which describes the slow evolution of pulses in a DM fiber.[2,3] In Ref. 3 the DMNLS equation was derived by use of the well-known technique of multiple scales (see also p. 89 of Ref. 4, for example), and its stationary solutions were computed numerically.

The derivation in Ref. 3 depends in a critical way on the accumulated dispersion profile's strict periodicity. However, in practice, fiber imperfections and other difficulties inevitably induce random fluctuations in various fiber parameters, destroying periodicity.[5,6] A natural question is whether a similar asymptotic theory can be developed for these aperiodic cases. In this Letter we investigate a class of fibers composed of individual spans whose dispersion values and lengths vary randomly about a specified ideal value, for which a systematic asymptotic analysis can be carried out. A typical profile of accumulated dispersion is depicted in Fig. 1. By adapting the method of multiple scales, we derive new averaged equations that are similar in form to the previously derived DMNLS equation. We demonstrate the validity of the multiple-scales technique with numerical simulations and also show the existence of new types of soliton, termed random DM (RDM) solitons, which are more resistant to the destabilizing effects of fiber randomness than the corresponding ideal DM soliton (DM soliton for the nonrandom, unperturbed fiber).

The governing nonlinear Schrödinger equation is

$$iE_{z'} + \frac{\beta_2(z')}{2} E_{tt} + \nu |E|^2 E = 0\,, \tag{1}$$

where $E(z', t)$ is the envelope of the electric field, $\beta_2(z')$ is the dispersion coefficient, and ν is the strength of the Kerr nonlinearity. The propagation direction is z', and the retarded time coordinate is t. We begin by transforming this equation into dimensionless variables. We normalize the variables in terms of characteristic parameters, denoted by the subscript $*$, $t = \tau/t*$, $z = z'/z*$, $u = E/\sqrt{P*}$, and $D = \beta_2/D*$, where $t*$ is the pulse width, $z*$ is the propagation distance, $P*$ is the characteristic power, and $D* = t*^2/|\beta_2|$ is the characteristic length of local dispersion. Typical normalizing parameters are $P* = 1$ mW, $z* = 1/(\nu P*) = 400$ km, $t* = 12$ ps, and $D* = 0.36$ ps^2/km, which correspond to a data transmission rate of 10 Gbits/s. If we include damping and amplification, our dimensionless equation is

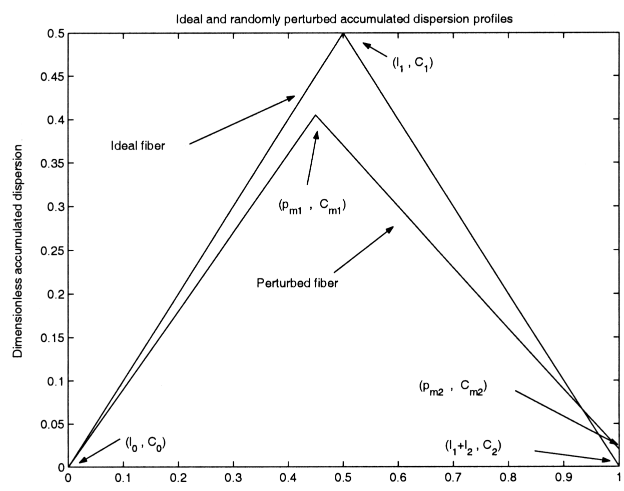

Fig. 1. Ideal and randomly perturbed accumulated dispersion profiles. The points (l_1, C_1), and $(l_1 + l_2, C_2)$ are coordinates corresponding to the ideal fiber, and (p_{m_i}, C_{m_i}), where $i = 1, 2$, are the coordinates corresponding to the perturbed fiber.

0146-9592/04/080821-03$15.00/0

$$i\,\frac{\partial u}{\partial z} + \frac{D(z)}{2}\,\frac{\partial^2 u}{\partial \tau^2} + g(z)\,|u|^2 u = 0\,. \tag{2}$$

In an ideal system the normalized GVD $D(z)$ and the damping and amplification coefficient $g(z)$ are assumed to vary periodically with period $z_a = l_a/z*$, where l_a is the amplifier spacing. This period is typically small when compared with the characteristic length scale of average dispersion ($z_a \sim 0.10$). We decompose the dispersion into a rapidly varying and average component $D(z) = [\tilde{D}(z/z_a)/z_a] + \langle D\rangle$, where $\langle D\rangle = \lim_{L\to\infty}\frac{1}{L}\int_0^L D(z')\,\mathrm{d}z'$. The presence of two evolution scales suggests an application of multiple scales. We introduce the variables $\xi = z/z_a$ and $Z = z$ so that $\partial/\partial_z = (1/z_a)(\partial/\partial\xi) + (\partial/\partial Z)$ and expand the solution in powers of z_a: $u(\tau, z; z_a) = u^{(0)}(\tau, \xi, Z) + z_a u^{(1)}(\tau, \xi, Z) + \cdots$. In the Fourier domain the solution at leading order is

$$\hat{u}^{(0)}(\omega, \xi, Z) = \hat{U}(Z, \omega)\exp[-iC(\xi)\omega^2/2], \tag{3}$$

where $\hat{U}(Z,\omega) = \int_{-\infty}^{\infty} U(Z,\tau)\exp(-i\omega\tau)\,\mathrm{d}\tau$ and $C(\xi) = \int_0^{\xi}\tilde{D}(\xi')\,\mathrm{d}\xi'$ is the Fourier transform of a slowly varying function that will be determined by the equation at the next order:

$$i\,\frac{\partial \hat{u}^{(1)}}{\partial \xi} - \omega^2\,\frac{\tilde{D}(\xi)}{2}\,\hat{u}^{(1)} = -R\{\hat{u}^{(0)}\}. \tag{4}$$

Here

$$R\{\hat{u}^{(0)}\} = i\hat{u}_z^{(0)} - \omega^2\,\frac{\langle D\rangle}{2}\,\hat{u}^{(0)} + N\{\hat{u}^{(0)}\},$$

where

$$N\{\hat{u}^{(0)}\} = \int_{-\infty}^{\infty}\exp(-i\omega\tau)g(\xi)\,|u^{(0)}|^2 u^{(0)}\,\mathrm{d}\tau\,.$$

After multiplication with an appropriate integrating factor, this equation can be written as

$$i\,\frac{\partial}{\partial\xi}\,[\hat{u}^{(1)}\exp\,iC(\xi)\omega^2/2] = [\exp\,iC(\xi)\omega^2/2]R\{\hat{u}^{(0)}\}\,. \tag{5}$$

To prevent secular growth, we must impose a solvability condition. If one considers the aperiodic function $C(\xi)$ as the limit of a periodic function with period approaching infinity, it follows that the averaged equation is

$$i\,\frac{\partial\hat{U}}{\partial Z} - \frac{\langle D\rangle}{2}\,\omega^2\hat{U} + \langle N\{\hat{U}\}\rangle = 0\,, \tag{6}$$

where

$$\langle N\{\hat{U}\}\rangle = \lim_{L\to\infty}\frac{1}{L}\int_0^L N\{\hat{u}^{(0)}\} \tag{7}$$

$$= \int_{-\infty}^{\infty}\mathrm{d}\omega_1\,\mathrm{d}\omega_2 K(\omega_1\omega_2)\hat{U}(\omega + \omega_1) \times \hat{U}(\omega + \omega_2)\hat{U}(\omega + \omega_1 + \omega_2)\,, \tag{8}$$

$$K(\Omega) = \frac{1}{4\pi^2}\lim_{L\to\infty}\frac{\int_0^L g(\xi')\exp[i\Omega C(\xi')/2]\mathrm{d}\xi'}{L}\,. \tag{9}$$

We note that this definition of the average is standard in the averaging theory developed for problems that lack periodicity.[4] For simplicity we consider the case of $g(z) = 1$.

We first describe the underlying ideal fiber in detail. We consider a two-step dispersion profile with GVD values that alternate in sign. If the GVD values within a span are labeled d_i, where $i = 1, 2$, and the lengths of each span are denoted l_i, where $i = 1, 2$, corresponding to points $\xi_i = \xi_{i-1} + l_{i-1}$ along the fiber, then the relationship $\sum_{i=1}^{2} d_i l_i = \langle D\rangle$ holds, where $\langle D\rangle$ is the average dispersion in the ideal case. Now consider the randomly perturbed case with GVD values in the mth span $\tilde{d}_{m_i} = d_{m_i} + \Delta d_{m_i}$, where $i = 1, 2$, and span lengths $\tilde{l}_{m_i} = l_{m_i} + \Delta l_{m_i}$, where $i = 1, 2$, corresponding to points along the fiber $\tilde{\xi}_{m_i} = \tilde{\xi}_{m_i-1} + \tilde{l}_{m_i-1}$. An ideal fiber and a typical perturbed fiber are depicted in Fig. 1. The accumulated dispersion at these points is thus $C(\tilde{\xi}_{m_i}) = C_{m_i} = \sum_{j=1}^{m}\tilde{d}_{j_i}\tilde{l}_{j_i}$. We compute the kernel function $K(\Omega)$ in Eq. (9) numerically, which yields a bounded continuous function of Ω.

Our averaging theory predicts that the solution should be close in an appropriate norm to the solution of Eq. (2) for long time scales $O(1/z_a)$. We demonstrate the validity of the asymptotic theory through numerical simulation for the cases of piecewise constant dispersion span values with random lengths and random dispersion span values with constant span lengths. Both parameters are taken to be uniformly distributed random variables with variations from a specified ideal value of 20%. The ideal values of local and average GVD are 1.8 and 0.036 ps/km^2 (normalized values of 5.0 and 0.1), respectively, and the ideal span length is 40 km (normalized value of 1.0). In each case a Monte Carlo simulation with 100 runs was performed, and the L^2 error $\int_{-\infty}^{\infty}|E|^2\,\mathrm{d}\omega$, where $E = \hat{u}(z,\omega)\exp[i\omega^2 C(z)/2] - \hat{U}(z,\omega)$ was computed. The error was then ensemble averaged over the number of fibers. The initial data for each simulation is an exact periodic solution for the averaged Eq. (6) computed by iteration. The results of our

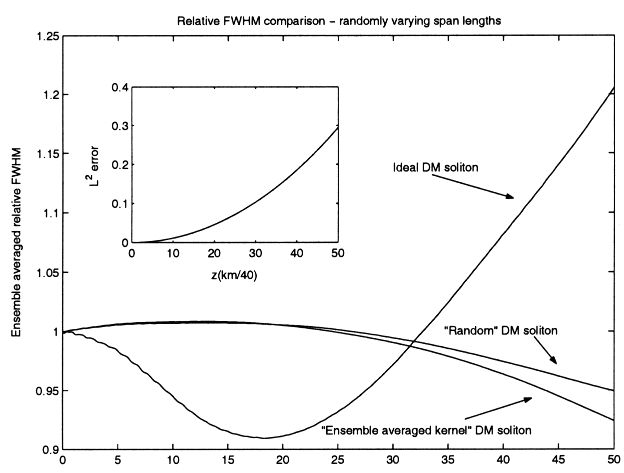

Fig. 2. Ensemble-averaged relative FWHM comparison of ideal, random, and ensemble-averaged kernel DM solitons for fibers with randomly varying span lengths with 20% deviation. Inset, L^2 error of averaging approximation.

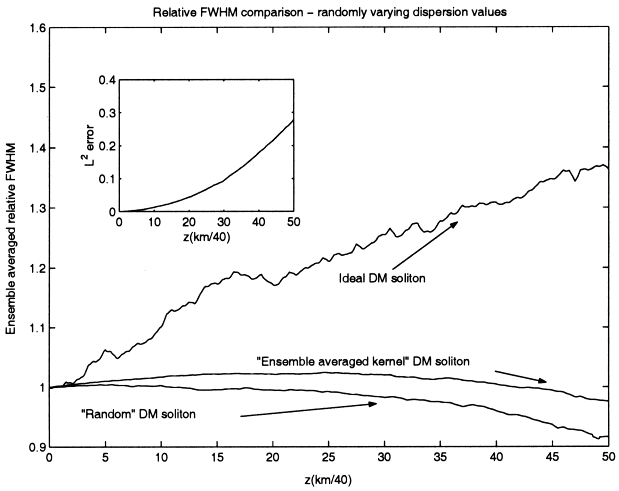

Fig. 3. Ensemble-averaged relative FWHM comparison of ideal, random, and ensemble-averaged kernel DM solitons for fibers with randomly varying dispersion values with 20% deviation. Inset, L^2 error of averaging approximation.

simulations show that for both randomly varying span lengths (Fig. 2 inset) and randomly varying dispersion values (Fig. 3 inset) the error in our approximations remains at worst $o(1)$ for time scales $O(1/z_a)$.

We now examine the dynamic properties of RDM solitons. It is well known that, in general, randomness in a fiber's chromatic dispersion profile will disrupt the balance of nonlinearity and dispersion required for soliton propagation (see Ref. 7 for a notable exception). This has been demonstrated numerically for the case of a DM fiber with white-noise randomness.[7,8] Also, interesting simulations with ideal DM solitons and noise similar to that depicted in Fig. 1 were carried out in Ref. 9.

Although pulse deterioration eventually takes place for the models that we studied, we find that, over the propagation scales considered, the RDM solitons are more robust to random perturbations than the corresponding ideal DM solitons. We compare the performance of the RDM solitons with the corresponding ideal DM solitons over the same fibers and furthermore study the performance of the ensemble–averaged kernel DM soliton, which is a stationary solution of Eq. (6) with the ensemble average of kernels $(1/N)\sum_{i=1}^{N} K^i(\Omega)$ replacing the individual $K^i(\Omega)$. For each fiber, all three types of soliton were used as initial data, and the FWHM of the pulses, normalized to their initial value at $z = 0$, was tracked. Our results indicate that for both the cases of randomly varying span lengths and dispersion values the FWHM of the ideal DM soliton eventually tends to grow with z, whereas the FWHM of both the RDM solitons and the ensemble-averaged kernel soliton are much more stable over the given propagation distance. The growth takes place more rapidly for fibers with randomly varying dispersion values than for fibers with randomly varying span lengths. This is shown for the case of randomly varying span lengths with 20% deviations in Fig. 2 and randomly varying dispersion values with 20% deviations in Fig. 3. These results demonstrate that even with random fiber lengths or dispersion values, there are pulse shapes that now play the role of fundamental solitons. Hence for a DM fiber with moderate noise the RDM soliton or ensemble-averaged kernel soliton, being in a sense better tuned to the collection of random fibers, are more stable in these random environments than the ideal DM soliton.

This work is partially supported by the National Science Foundation under grant numbers ECS-9800152, DMS-0070792, and DMS-9810751. J. Moeser's e-mail address is moeser@colorado.edu.

References

1. C. Lin, H. Kogelnik, and L. G. Cohen, Opt. Lett. **5,** 476 (1980).
2. I. Gabitov, E. G. Shapiro, and S. K. Turytsin, Phys. Rev. E **55,** 3624 (1995).
3. M. J. Ablowitz and G. Biondini, Opt. Lett. **23,** 1668 (1998).
4. J. A. Sanders and F. Verhulst, *Averaging Methods in Nonlinear Dynamical Systems* (Springer-Verlag, New York, 1985), p. 36.
5. L. F. Mollenauer, P. V. Mamyshev, and M. J. Neubelt, Opt. Lett. **21,** 1724 (1996).
6. P. K. A. Wai, C. R. Menyuk, H. H. Chen, and Y. C. Lee, IEEE J. Quantum Electron. **24,** 373 (1988).
7. M. Chertkov, I. Gabitov, and J. Moeser, Proc. Natl. Acad. Sci. USA **98,** 14208 (2001).
8. B. A. Malomed and A. Berntson, J. Opt. Soc. Am. B **18,** 1243 (2001).
9. C. J. Xie, L. F. Mollenauer, and N. Mamyshev, J. Lightwave Technol. **21,** 769 (2003).